Setting Dead at Zero? On the contingency of the utility unit scale

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Abstract

In the valuation of health (states), *dead* is usually conceptualised as a state with a utility value of zero. Here I argue that, despite its wide adoption, a sound theoretical basis for this practice is lacking. The development of alternative value frameworks, without *dead* as an anchor point on the utility scale, may well be permissible.

Background

In the QALY model, health state utilities are commonly measured on a scale that is anchored at full health, set to a value of one, and dead, set to zero. Even though this practice is adopted almost universally, a theoretical basis for it appears to be missing.[1] In their 2018 literature review, Roudijk et al.[2] report that most authors do not justify setting dead to zero, and even those who do, merely state that this is done 'by definition', 'by convention', or simply, 'for convenience'. Notwithstanding, some authors have proposed theoretical arguments for why dead has to be set to zero and not to any other value. In this brief report, I revisit and rebut the four arguments known to the authors, to demonstrate that anchoring the utility scale on a different state may well be permissible.

Argument 1: Utilities are measured on a ratio scale, with dead as a natural zero point

The first argument is taken from Roudijk et al.[2] They state that, in order for the QALY model to satisfy basic principles of rationality, utilities must be measured on a ratio scale, for which dead is as a natural zero point.

Let's consider the first part: is dead a natural zero point? For physical quantities, such as mass or temperature, zero has an unambiguous meaning. It defines the point at which there is no mass or no molecular motion. Yet, the position of dead on the utility scale seems to have a different function. Most

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people consider certain states worse than dead, so dead does not mark the state with the lowest utility value. Instead, it divides the scale into states with positive (better than dead) and negative values (worse than dead). This seems incompatible with the notion of a natural zero, like in 0° Kelvin (= absence of molecular motion), but rather appears to be a zero with an arbitrary reference, like in 0° degrees Celsius or 0° Fahrenheit (= freezing point of water or salty water).

Returning to the first part of the argument, must utilities be measured on a ratio scale? For an interval scale to be admissible as the basis for the QALY model, preferences must be invariant to positive affine transformations, i.e. $f(A) \sim f(B) \rightarrow f'(A) \sim f'(B)$, for any $f'(x) = a * f(x) + b$).[3,4] This is a basic rationality requirement, and, if Roudijk et al.[2] were right in that it is violated, the interval scale may be inappropriate to measure health state utilities (see Table 1 for an overview of measurement scales and their properties). In the following, we reproduce the example given in their paper (with minor adaptations) and show that their reasoning is flawed.

The QALY model is represented mathematically as: $Q = u(h_i) * v(t_i)$, whereby $u(h_i)$ denotes the utility derived from state i, and $v(t_i)$ is a function of the time spent in that state – note that in the standard model $v(t) = t$. Now, suppose Alice is indifferent between living 10 years in full health with $u(h_{full}) = 1$, followed by 10 years in state i, with $u(h_i) = 0.7$ (**A**), and living 15 years in full health, followed by five years in state j, with $u(h_j) = 0.4$ (**B**). Both options yield 17 QALYs, as shown in equations A.1 and B.1 below.

$$A.1: \quad 1 * 10 + 0.7 * 10 = 17$$
$$B.1: \quad 1 * 15 + 0.4 * 5 = 17$$

To show that a positive affine transformation (i.e. $f'(x) = a * f(x) + b$) does not preserve indifferences, Roudijk et al.[2] shift the origin of v(t), with $v'(t) = t + 2$, and derive the following result:

$$A.2: \quad 1 * (10 + 2) + 0.7 * (10 + 2) = 20.4$$
$$B.2: \quad 1 * (15 + 2) + 0.4 * (5 + 2) = 19.8$$

Table 1: Measurement scales and their properties

Scale	**Description**	**Examples**	**Invariance**
Nominal	Qualitative classification	Blood groups, gender	n.a.
Ordinal	Rank order, distances between ranks are not known or nor defined	Likert scale, quantile rankings	$u(x)^2$, $ln(u(x))$
Interval	Ranking with meaningful differences, ratios of values and the zero point have no meaning (20°C is 15° warmer than 5°C, but it is not four-times warmer)	°C, °F, vNM utilities	$3x + 4$
Ratio	most informative scale; order, differences, ratios, and the zero point are meaningful	Meters, gramm, °Kelvin	$3x$

Alice now appears to prefer A over B, and Roudijk et al. conclude that an interval scale is inadmissible as a basis for the QALY. Therefore, they argue, utilities must lie on a ratio scale. Yet, their algebra is flawed: note that in A, Alice spends zero time in state j. In the standard model, any state in which zero time is spent can be omitted from the equation (because $u(h_j) * v(t_j) = 0.4 * 0 = 0$). However, after the origin is shifted, this is no longer allowed, because now $v'(0) = 2$. State j must thus be considered in A, as must state i in B. When the positive affine transformation is applied consistently, Alice's transformed utility function is represented by the following equations:

$$A.3: \quad 1 * (10 + 2) + 0.7 * (10 + 2) + 0.4 * (0 + 2) = 21.2$$
$$B.3: \quad 1 * (15 + 2) + 0.4 * (5 + 2) + 0.7 * (0 + 2) = 21.2$$

Alice's indifference is well preserved, and a compelling reason for rejecting the interval scale as a basis for the QALY model cannot be found.

Argument 2: Dead and zero time indifference

Another interesting mathematical argument for setting dead to zero has been derived from Miyamoto et al.'s[5] seminal work on 'the zero-condition': it is maintained that all health states are equally preferred when their duration is 0, i.e. $u(h_i) * v(0) = u(h_j) * v(0)$ for any states i and j. Taking this a step further, Roudijk et al.[2] claim that indifference should also hold for a choice between being in state i for a duration of 0 (followed by death) and being dead for some time t, i.e. $u(h_{dead}) * v(t) = u(h_i) * v(0)$. For this equation to hold true – for any duration t and any state i – the utility of dead must be zero: $u(h_{dead}) = 0$.

While this may sound logical at first, the premise of the argument appears dubious. The presented alternatives are not mutually exclusive. In fact, they are identical: being dead for some time t involves spending zero time in state i. Which also involves spending zero time in state j. One does not have to forgo one for the other. But preferences can only be meaningfully specified, if there is a choice involved. It thus seems impossible to postulate any indifferences here, and insights about the value of dead can not be derived.

Argument 3: Streams of infinite non-zero utilities

The third argument comes from the third edition of Drummond et al's[6] standard textbook 'Methods for the Economic Evaluation of Health Care Programmes' (not included in the 4^{th} edition):

> *"[I]f any score other than zero were used for death, [...] the (non-zero) death score would be assigned to the state of death for each year off into the future for as long as the dead lasted (that is, forever). Thus, the analyses would have streams of numeric outcomes going to infinity - not a pretty picture. Accordingly, zero is the only practical score that can be used for death."*

First of all, it should be noted that an infinite stream of zero utility values is also undesirable, because, strictly speaking, the product of zero and infinity is not defined. However, the concerns about infinite streams of non-zero utilities are unfounded. Future utilities are usually discounted, which causes non-zero utilities to tend towards zero. This prevents any infinite values from occurring.

But even without discounting, infinite streams of non-zero utilities from being dead are not a problem, because they occur in all alternatives. Since economic evaluations are only concerned with the differences between alternatives (the increments), those non-zero utilities cancel out and can be discarded.

Argument 4: The ontological zero

The fourth and final argument is less mathematical and more ontological. It states that once you are dead, you cannot experience utilities anymore and, thus, zero is the only plausible value for dead.[7]

The argument is problematic for two reasons. Firstly, it conflates disparate concepts of utilities: the notion of (absolute) experienced utility used here for the dead state might be incompatible with the (relative) decision utilities measured for any other health state. Secondly, the argument might also be objectionably paternalistic. There are many different ideas of death. When people value the dead state using a visual analogue scale (0-100), their responses, unsurprisingly, vary. For example, one participant in an EQ-5D health valuation study commented that dead can have two values, 100 for 'dead in heaven', and 0, for 'dead in hell'.[8] It would be presumptuous to discard this and other beliefs about death, and to simply assume that everyone shares a supposedly scientific zero utility valuation of dead.

Why does the value of dead matter?

Social values for health states are commonly derived by averaging over the utilities from different individuals. For this operation to be permissible, units of utilities must be measured on the same scale and interpersonally comparable.[4,9] To use an analogy, it would be meaningless to take the average of a range of temperature measurements, some in °Kelvin, others in °Celsius, and still others in °Fahrenheit. For the valuation of health, a common unit scale is enforced by anchoring everyone's utilities at two points; by convention, full health and dead. The distance between those two is assumed to be the same for everyone and comparable across individuals.

It is not within the scope of this paper to further discuss the intricate problem of interpersonal utility comparisons, but it should be noted that, depending on which anchor points are chosen, aggregate social value sets may differ.[4,10] Therefore, the anchor points matter, and should not be assigned arbitrarily. While the use of full health as the upper anchor point seems indisputable - it is a dominant state, which should be weakly preferred over all other states -, it seems to be a matter of debate whether dead is an (or the only) appropriate lower anchor point. At least the four arguments considered in this paper fail to provide an unequivocal basis for setting dead to zero.

The results of this paper do not imply that dead must not be used as an anchor point, yet, they suggest that dead may not have to be used as one. Relaxing this property of the QALY model may open up new possibilities to develop alternative value frameworks and re-consider the role of states worse than dead. In any case, an open, impartial discussion about appropriate utility scales may be more valuable than trying to (ex-post) justify pragmatic decisions, taken decades ago, in the early days of the field. Even if dead is to be kept as an 'absolute zero', this work may have highlighted a weakness in the conceptual foundations of health economics, and, hopefully, it sparks more interest in this topic.

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